# PROCESS CHARACTERISTICS OF SCREW IMPELLERS WITH A DRAUGHT TUBE FOR NEWTONIAN LIQUIDS. THE POWER INPUT

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An expression has been proposed for the power input of a screw impeller with a draught tube in the creeping flow regime based on the analogy with extruder screws. Experimental verification has confirmed practical utilita of the expression in a wide range of geometrical parameters of the  $\cdot$ impeller and for the Reynolds number for mixing below 20. The total power input of the impeller is expressed as a sum of the input inducing the drag flow and the input to create the pressure flow. The former of the inputs may be deduced from the theory of extruders while an empirical approach based on experiment has been used to formulate an expression for the latter.

The screw impeller with a draught tube is one of two principal types of impellers suitable for mixing highly viscous liquids. An important quantity necessary for the design of mixing equipment is the power input of the impeller. This work has been devoted to finding a suitable relationship for the calculation of the power input of the screw impeller.

The problem of the relation between the power input of the screw impeller and principal physical and geometrical quantities has been investigated in relatively many papers<sup> $1-8$ </sup>. The results of these works have proven that the power input of this type of impeller in the creeping flow regime is significantly affected by the geometrical simplexes of the mixing system.

In search for a fundamental relationship for the power input of the impeller one can use of the so-called inspection analysis of the basis equations. Thus we start from the following basic equation<sup>9,19</sup>

$$
P = \oint_{S} \mathbf{v} (\tau - p\delta) \, \mathrm{d}S \tag{1}
$$

and the Navier-Stokes equation<sup>8</sup>

$$
\varrho \, \frac{\mathbf{D} \mathbf{v}}{\mathbf{D} t} = -\nabla p + \mu \, \nabla^2 \mathbf{v} + \varrho \mathbf{g} \, . \tag{2}
$$

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Considering the Newtonian liquid<sup>8</sup>

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$$
\tau = \mu \Delta \tag{3}
$$

the viscosity of the liquid,  $\mu$ , is a scalar function of temperature and pressure. Using quantities specifying the mixing system, n and  $d$ , Eqs  $(1)$  and  $(2)$  may be rendered dimensionless

$$
\mathbf{v}^* = \frac{\mathbf{v}}{nd} \qquad \mathbf{g}^* = \frac{\mathbf{g}}{g}
$$
  

$$
A^* = \frac{A}{n} \qquad \nabla^{*2} = d^2 \nabla
$$
  

$$
\mathbf{S}^* = \frac{\mathbf{S}}{d^2} \qquad p^* = \frac{p}{\mu n}
$$
  

$$
\nabla^* = d \nabla.
$$
 (4)

For stationary conditions and for the creeping flow regime *(i.e.* neglecting the effect of gravitational forces) one can obtain the following modified dimensionless equations<sup>19</sup>

$$
P|\mu n^2 d^3 = \oint_S \mathbf{v}^* (A^* - p^* \delta) \, \mathrm{d} \mathbf{S}^* \tag{5}
$$

$$
\frac{\mu}{nd^2\varrho}\mathbf{v}^*\nabla^*\mathbf{v} = -\nabla^*p^* + \nabla^{*2}\mathbf{v}^*.
$$
 (6)

**In** the creeping flow region the Reynolds number for mixing, Re

$$
Re = nd^2 \varrho / \mu \tag{7}
$$

takes low values. Accordingly, the quantities  $p^*$ ,  $\mathbf{v}^*$ , and as  $\Delta^*$  are functions of position only. Based on this analysis one can write for the dimensionless power input *p\**  in creeping flow region

$$
P^* = f \text{ (the geometrical simplexes of the system)} \tag{8}
$$

while

$$
P^* = P/\mu n^2 d^3 \tag{9}
$$

At the same time we have

$$
P^* = \text{Po} \cdot \text{Re} \,. \tag{10}
$$

Here Po is the power input criterion

$$
Po = P \mid qn^3d^5 \tag{11}
$$

In the solution of the effect of the geometrical configuration on the power input of the screw impeller with a draught tube one can use, to some extent, the similarity of this equipment with the extruding screw  $-$  see Fig. 1. Between both systems though there is a basic difference as to their function: The mixing screw is supposed to maximize pumping at low pressure losses, while the extruding screw maximizes pressure at low pumping capacity. This is also the reason for the difference in the depth of the screw channel,  $h_k$ , and the overall length of the screw,  $h_k$ . The expressions from the theory of the power input of extruding screws must be therefore modified by a correction factor, especially for the depth of the screw channel.

The above assumption is met by the derivation of the power input presented by Bernhardt<sup>10,20</sup> after incorporating Squires' correction factor. The resulting expression in the dimensionless form is given by Eq.  $(8)$  (ref.<sup>11</sup>)

$$
P^* = P_{\rm D}^* + P_{\rm P}^* \,. \tag{12}
$$



### FIG. 1

A: Mixing equipment with the screw impeller and a draught tube. V mixing vessel, D draught tube, M screw impeller. B: Sketch of the thread and the jacket of an extruder. B jacket, S extruding screw

The dimensionless power input of the screw  $P^*$  is given as a sum of the dimensionless power input needed to induce the drag flow  $P_p^*$  and the dimensionless power input to create the pressure flow,  $P_{p}^{*}$ .

According to the relationships published by Bernhardt<sup>10</sup>, the power input  $P_p$ may be computed for the creeping flow regime from these equations

$$
P_{\rm D} = P_{\rm D}^* \mu n^2 d^3 \tag{13}
$$

$$
P_{\rm D} = P_{\rm D}^* \mu n^2 d^3 \qquad (13)
$$

$$
P_{\rm D}^* = 2\pi^3 \left(\frac{h_{\rm D}}{h_{\rm v}}\right) \left(\frac{h_{\rm v}}{d}\right) \left(\frac{D'}{d}\right) \left[\frac{d}{D' - d_0} \frac{s - e}{s} F_z \left(\cos^2 \phi + 4\sin^2 \phi\right) + \frac{d}{D' - d} \frac{e}{s}\right]. \qquad (14)
$$

The meaning of individual symbols is apparent from Fig. 1.  $F<sub>z</sub>$  represents the power input correction factor, dependent, according to Squires<sup>20</sup>, on the ratio of the depth,  $h_k$ , and the width, w, of the screw channel. Its value can be read off the diagram in Fig. 2.





Dependence of the correction factor F*<sup>z</sup>* on the relative depth of the screw channel  $h_k/w$  according to Squires<sup>19,20</sup>

$$
x = \frac{h_{k}}{w} = \frac{D' - d_{0}}{2(s - e) \cos \varphi}
$$





. Scheme of experimental arrangement to measure impeller power input. H hydromotor, I magnetic impulse transducer, R transistor relay,  $C$  impulse counter,  $S_1$  impeller shaft, M screw impeller, D draught tube, V mixing vessel,  $T_1$  turntable,  $T_2$  hydraulic bench, S desk balance,  $V_1$  pressure air valve,  $V_2$ pressure oil valve

The power input corresponding to the pressure flow is given in dimensionless form  $as^{10,12}$ 

$$
P_P^* = \Delta p \dot{V}_{\text{max}} / \mu n^2 d^3 \,. \tag{15}
$$

In this expression  $\Delta p$  represents pressure drop due to recirculation of a highly viscous liquid in a system with a screw impeller, or the pressure loss in the extruder jet.  $\dot{V}_{\text{max}}$  is the maximum pumping capacity of the given screw, the so-called drag flow in the screw (corresponds to zero pressure drop).

While the magnitude of  $\dot{V}_{\text{max}}$  depends on the geometry of the screw only,  $\Delta p$  is affected not only by the screw geometry but also by the geometrical arrangement of the whole system. The theoretical way of determining  $\Delta p$  has not been possible owing to the complexity of the mixing system.

The effect of the geometrical simplexes of the system with the screw impeller and the draught tube on the power input *P;* 

$$
P_{\mathbf{P}}^* = f\left(\frac{D}{d}, \frac{S}{d}, \frac{d_0}{d}, \frac{h_v}{d}, \ldots\right) \tag{16}
$$

has to be therefore determined experimentally.

### EXPERIMENTAL

The measurements were carried out in vertical cylindrical vessels with flat bottom, 290, 390 and 395 mm in diameter. In the axis of the vessel there was a screw impeller, in a total of 11 modifications differing one from another by the size and/or some geometrical simplexes of the screw. The dimensions of the screws used are summarized in Table I.

### TABLE I

Dimensions of the screw impellers, mm



<sup>a</sup> Lead of the seven impellers No 2 was as follows:



The diameter of the draught tube was in all cases chosen in accordance with the standard ON 691 028 (ref.<sup>13</sup>). The impellers were located always so that the height of the impeller above the bottom equalled the height of liquid above the impeller.

The shaft of the impeller was powered by a hydromotor with a continuous control of the frequency of revolution in the range between 10 and 1 000. The rpm of the impeller were measured by a magnetic impulse detector S 585-ZPA. The impulses were transmitted *via* a transistor relay R 585-ZPA to TESLA BM 445 E counter to be counted in a pre-selected time interval.

The mixing vessel was placed on a turntable, the passive resistance of which was minimized by buoying the table by pressurized air. The torque produced by the rotating impeller was transmitted to a silon thread and compensated on a desk balance. The whole experimental set up is shown schematically in Fig. 3.

The magnitude of the force, and hence also the range of the torque, could be varied on the one hand by altering the lever of the force  $(r = 0.59$  and  $0.21$  m), and, on the other hand, by changing the leverage ( $p = 1$  or 2.032). The mass of the weight G for each measurement was taken as the mean of the weights after forcing the balance pointer to the left and right (to eliminate passive resistances).

The magnitude of the torque was determined from the relation

$$
M = p'r'Gg \tag{17}
$$

and the appropriate impeller power input *P* 

$$
P = 2\pi n M. \tag{18}
$$

As model liquids we used solutions of starch syrup in water, which exhibit Newtonian behaviour. The density  $\varrho$  of the model liquids was measured by pycnometry, the dynamic viscosity  $\mu$ on a rotational viscometer Rheotest R V.

### RESULTS

Using the measured values of the power input,  $P$ , the frequency of revolution,  $n$ , and the physical and geometrical quantities of the mixed and the mixing system, the values of the criteria Po and Re were evaluated. Simultaneously also their product, *P\*,* was computed.

Graphical representation of the functions Po =  $f (Re)$  is shown Figs 4-6.

From the experimental results it followed that for each individual experimental configuration of the screw impellers and the draguht tube, for  $Re < 20$ , one may assume, that

$$
P^* = \text{Po Re} = \text{const.} \tag{19}
$$

Taking into account the condition of the creeping flow in the mixed liquid, mean values of the quantity  $P^*$  and its variance,  $S_p$ , were computed from measured  $P^*$ , together with the 95% confidence limits  $\pm z$ . The found values for individual experimental runs are summarized in Table **II.** 

In order to evaluate the effect of geometrical configuration on the magnitude of the dimensionless power input for the pressure flow  $P_p^*$ , it was necessary to calculate values of the dimensionless power input  $P_D^*$  expended to induce the conveying flow within the draught tube. Assuming the existence of the analogy between the mixing and the extruding screw the magnitude of  $P_D^*$  was computed for individual configurations from Eq. (14).

In the dimensionless form the power input  $P<sub>p</sub>$  can be evaluated as the difference of the overall power input P and the input  $P_{\text{D}}^*$ .

$$
P_p^* = P^* - P_D^* \tag{20}
$$

As has been already mentioned in the theoretical part the difference  $(P^* - P_D^*)$ depends on the geometrical configuration. From Table II it follows that the considered effect of the geometry of the system was represented through the simplexes  $D/d$ , s/d and  $d_0/d$  and in a small range also by  $h_v/d$ . Excepting the impeller number 5, the diameter of the core of the screw,  $d_0$ , was the same for all impellers. The value of the simplex *do/d* was to some extent (change of *D)* dependent on the value of the





Power input dependence  $Po = f(Re)$  for screw impellers with a draught tube. 1 Screw No 2F  $D/d = 2.30 \, (\odot)$ ; 2 screw No 2G  $D/d =$  $= 2.30$  ( $\bullet$ ); 3 screw No 2D  $D/d = 2.30$  ( $\circ$ ); 4 screw No 2A  $D/d = 2.30$  ( $\circ$ ); 5 screw No 5  $D/d = 1.98$  (0)



 $0.1$ 

 $10$ 



 $10$ 

Power input dependence  $Po = f(Re)$  for screw impellers with a draught tube. 1 Screw No 1  $D/d = 3.37$  ( $\otimes$ ); 2 screw No 2E  $D/d =$  $=4.10$  (O); 3 screw No 3  $D/d = 2.69$  ( $\bullet$ ); 4 screw No 4  $D/d = 2.13$  (0); 5 screw No 4  $D/d = 1.59$  ( $\odot$ )

simplex  $D/d$ . The resulting correlation in the form

$$
\frac{\overline{P}^* - P_D^*}{h_v/d} = k_1 \left(\frac{D}{d}\right)^{a_1} \left(\frac{s}{d}\right)^{b_1} \left(\frac{d_0}{d}\right)^{c_1}
$$
\n(21)

### TABLE II

Results of power input measurements for screw impellers in the creeping flow regime ( $Re < 20$ ),  $h_v/d = 1.5$ ,  $D'/d = 1.1$ 



TABLE **III** 

Dimensionless power input  $P_D^*$  inducing drag flow in the draught tube



was therefore formulated also from mean values of  $\overline{P}$  from the work of Reiger and Nová $k^{3,14}$ . In these cited papers attention has been paid expecially to the effect of the simplex  $d_0/d$  on the magnitude of the power input of the screw impeller

### TABLE IV

Regression analysis of power input measurements





FIG. 6

Power input dependence  $Po = f(Re)$  for screw impellers with a draught tube. 1 Screw No 4  $D/d = 2.12 \ (\oplus)$ ; 2 screw No 2C  $D/d =$  $= 2.30$  ( $\bullet$ ); 3 screw No 2G  $D/d = 3.10$  ( $\circ$ ); 4 screw No 2B  $D/d = 2.30$  ( $\otimes$ ); 5 screw No 2A  $D/d = 3.10$  (0); 6 screw No 3  $D/d =$  $2.00$  ( $\circledcirc$ )





A comparison of the calculated (Eq. (22) and the experimental values of  $\overline{P^*}$  in the creeping flow region. Screws with  $D/d < 2$ (0); screw with  $2 \le D/d \le 2.5$  (8); screws with  $D/d > 2.5$  ( $\bullet$ )

with a draught tube. Principal values of the simplexes  $D'/d$  and  $h<sub>v</sub>/d$ , as well as the position of the impeller in the vessel were the same in those papers<sup>3,14</sup> as in this work.

The effect of the simplex  $(h_v/d)$  was taken for the given small interval to be linear. Computed values of  $P_p^*$  for the screws used in this work are summarized in Table III.

To evaluate the coefficients of Eq.  $(21)$ , all experimental configurations have been considered (those presented here, as well as those from papers<sup>3,14</sup> for which  $\bar{P}^* - P_D^*$ ) > 0). The results of statistical processing are presented in Table IV.

The results of the statistical processing suggest that all factors considered are statistically significant at the selected level of significance of  $5\%$  (and even on the  $0.5\%$  level). The variance and the correlation coefficient are within acceptable limits.

The power input of a screw impeller with a draught tube in the creeping flow region thus can be assessed from the following relationship

$$
\frac{P}{\mu n^2 d^3} = P_{\rm D}^* + 25.9 \left(\frac{D}{d}\right)^{-4.39} \left(\frac{s}{d}\right)^{1.71} \left(\frac{d_0}{d}\right)^{-1.70} \left(\frac{h_{\rm v}}{d}\right). \tag{22}
$$

The region of validity of the above expression is as follows Re < 20; 1.58  $\leq D/d \leq$  $\leq$  3.94;  $0.33 \leq s/d \leq 1.50$ ;  $0.17 \leq d_0/d \leq 0.60$ ;  $1.37 \leq h_v/d \leq 1.50$ ; for these con-

### TABLE V

A comparison of the power input measurements from this work with the literature data for the creeping flow regime



stant geometrical simplexes;  $H/D = 1.0$ ;  $D'/d = 1.1$ ;  $h_D/D' = 1.15$ ;  $H_2/d = 0.5$  $(D/d - 1.5)$ .

Analysis of Eq. (22) suggests that the power input for the pressure flow increases with diminishing *Did* ratio (increasing diameter of the screw; with increasing *sid* ratio (increasing lead of the screw); with decreasing  $d_0/D$  ratio (diminishing size of the core of the screw) and it may be assumed that also with increasing  $h<sub>v</sub>/d$  ratio (increased total height of the screw).

Computed values (Eq\_ (22)) of the dimensionless power input of the screw impeller P for all experimental configurations from this paper as well as those from the literature<sup>3,14</sup> were compared with the experimentally found mean values of  $\bar{P}^*$  in the correlation diagram (Fig. 7). It turns out that the agreement of the computed and experimental values is very good.

### DISCUSSION

Experimental values of the mean value of the criterion  $P^*$  found in this work may be compared with analogous data from literature\_ This comparison is furnished in Table V.

Based on data in Table V it may be concluded that the results of measurement of the power input of screw impellers from this work agree:

- very well with the measurement of Rieger<sup>14</sup> and Novák<sup>3</sup> (the used screws were very similar in shape with the those from this work)
- relatively well with the results of Nagata<sup>1</sup> and  $Ho^{16}$



### TABLE VI

A comparison of  $\overline{P}^*$  and  $P_D^*$  (Eq. (14)) for very low  $\Delta p$ 

- insufficiently with the results of Prokopec<sup>7</sup> and Serwinski<sup>5</sup> and Hoogendoorn<sup>2</sup> (not presented in Table V), which were obtained by measuring substantially longer screws and on a somewhat different geometrical configuration.

This analysis shows that the measured values of power input are acceptable and that the measurements were not subjected to a systematic error.

The satisfactory accuracy of the estimate of values of  $P^*$  (Table II) was ensured on the one hand by sufficient sensitivity of the measuring system, and, on the other hand, by the sufficient number of experimental data.

The slope of the dependence Po =  $f(Re)$  in the log-log coordinates (Fig. 4-6). in the laminar flow region may be regarded with sufficient precision, to be  $-1$ . This value is in accord with the theoretical and experimental results of the cited papers<sup>1-7,14,16</sup>.

In order to find out the effect of the geometrical simplexes of the screw impeller, the draught tube and the surrounding system the correlation (22) was proposed. This correlation is based essentially on the theory of extruding screws.

The value of  $P_{\rm D}^*$ , corresponding to the power input used to induce the drag flow, is fairly strongly dependent on the simplexes of the screw,  $s/d$  and  $d_0/d$ , and partially also on  $D/d^{10}$ . This is apparent from Eq. (14). Because

$$
P_{\rm D}^* = P^* - \frac{\Delta p \dot{V}_{\rm max}}{\mu n^2 d^3} \tag{23}
$$

the experimental values of  $P^*$  should approach  $P^*_{\text{D}}$  for those configurations for which  $\Delta p \rightarrow 0$ . This is the case if the screw

- $-$  is relatively small (large value of  $D/d$ )
- has a small lead (low value of  $s/d$  small pumping effects of the screw)
- $-$  has a relatively large diameter core (large value of  $d_0/d$ , small pumping effects of the screw).

Table VI gives the comparison of  $P^*$  and  $P_D^*$  (Eq. (10)) for the above configurations.

The difference between the dimensionless power inputs  $P^*$  and  $P^*_{\rm D}$  at low pressure drop in the recirculation (for systems with a screw impeller) is thus very small indeed. In one case in fact, the experimental value of  $P^*$  - the total power input - is lower than the computed value of  $P_{\rm p}^*$ . The use of the theory of extruding screws to calculate the power input  $P_D^*$  inducing the drag flow, may thus be regarded as justified.

The effect of the geometrical arrangement of the mixing system on the power input bringing about the pressure flow is given by

$$
P_{\rm P}^* \sim \left(\frac{D}{d}\right)^{-4.39} \left(\frac{s}{d}\right)^{1.71} \left(\frac{d_0}{d}\right)^{-1.70}.
$$

This part of the total power input is markedly dependent on the pressure drop in the recirculation  $-\Delta p$ . This is the reason for the strong effect of the relative increase of size of the impeller in the mixing vessel *(Did).* 

Practically only one work offers the chance of comparing the effect of the geometrical arrangement of the system on the power input inducing the pressure flow. According to Novák<sup>3</sup>

$$
P_{\rm P}^* \sim \left(\frac{D}{d}\right)^{-4.54}
$$

which agrees quite well with the results of this work.

The effect of the simplexes  $s/d$  and  $d_0/d$  on the power input  $P_p^*$  was not found in the available literature. The papers of Prokopec<sup>7</sup>, Serwinski<sup>5</sup> and Chavan<sup>17</sup> examine the effect on the total power input of *sid* only.

### LIST OF SYMBOLS



### Seichter, Dohnal, Rieger



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